Effect of damping on inelastic response spectra

Ghazi J. Al-Sulaimani
University of Petroleum and Minerals, Dhahran, Saudi Arabia
Jose M. Roesset
University of Texas, Austin, USA

ABSTRACT: The effect of initial damping on inelastic spectra is investigated for elasto-plastic systems. It is found that Newmark's rules to construct inelastic spectra from the elastic ones are appropriate for systems with 5% damping. As the damping increases, however, the reduction in acceleration with ductility ratio tends to decrease. Conversely this reduction is higher than that predicted by Newmark's rules for smaller values of damping. The results presented allow to derive smooth spectra for desired values of damping and ductility from an elastic design spectrum.

1 INTRODUCTION

The usefulness of response spectra as a means to characterize an earthquake and its effects on a structure was recognized very early. For some time, however, designers were using in every case the set of response spectra corresponding to the 1940 El Centro earthquake. A first step to improve this situation was taken by Housner (1959). By considering the records of five strong motion earthquakes, Housner started by defining a new measure of intensity, consisting of the area under the velocity response spectrum for 20% of critical damping from 0.1 to 2.5 seconds. He scaled then the corresponding records so that they would have the same intensity and averaged the corresponding response spectra. The resulting set of smooth, average spectra was used extensively for some years. A second step towards a better characterization of strong motions for design purposes was taken by Newmark (1967) recognizing seven distinct ranges in a response spectrum (see Figure 1):

1. A range of very small frequencies for which the relative displacement of a single degree of freedom (SDOF) is practically equal to the maximum ground displacement.
2. A transition range.
3. A range of frequencies over which the relative displacement of the SDOF system reaches a maximum value and remains practically constant in the average.
4. A range over which the relative velocity (or pseudo-velocity) of the SDOF system is practically constant, with again its maximum value.
5. A range over which the absolute acceleration (or pseudo acceleration) of the system is almost constant in the average, with its maximum value.
6. A second transition zone.
7. A range of high frequencies for which the absolute acceleration (or pseudo acceleration) of the SDOF system is practically equal to the maximum ground acceleration.

Of these seven ranges the intermediate ones (3, 4 and 5) and particularly the 4th and 5th ranges are the ones of greatest interest for most structures. They correspond to the maximum spectral displacement, velocity and acceleration and they will be referred to in this paper as the displacement, velocity and acceleration ranges in short.

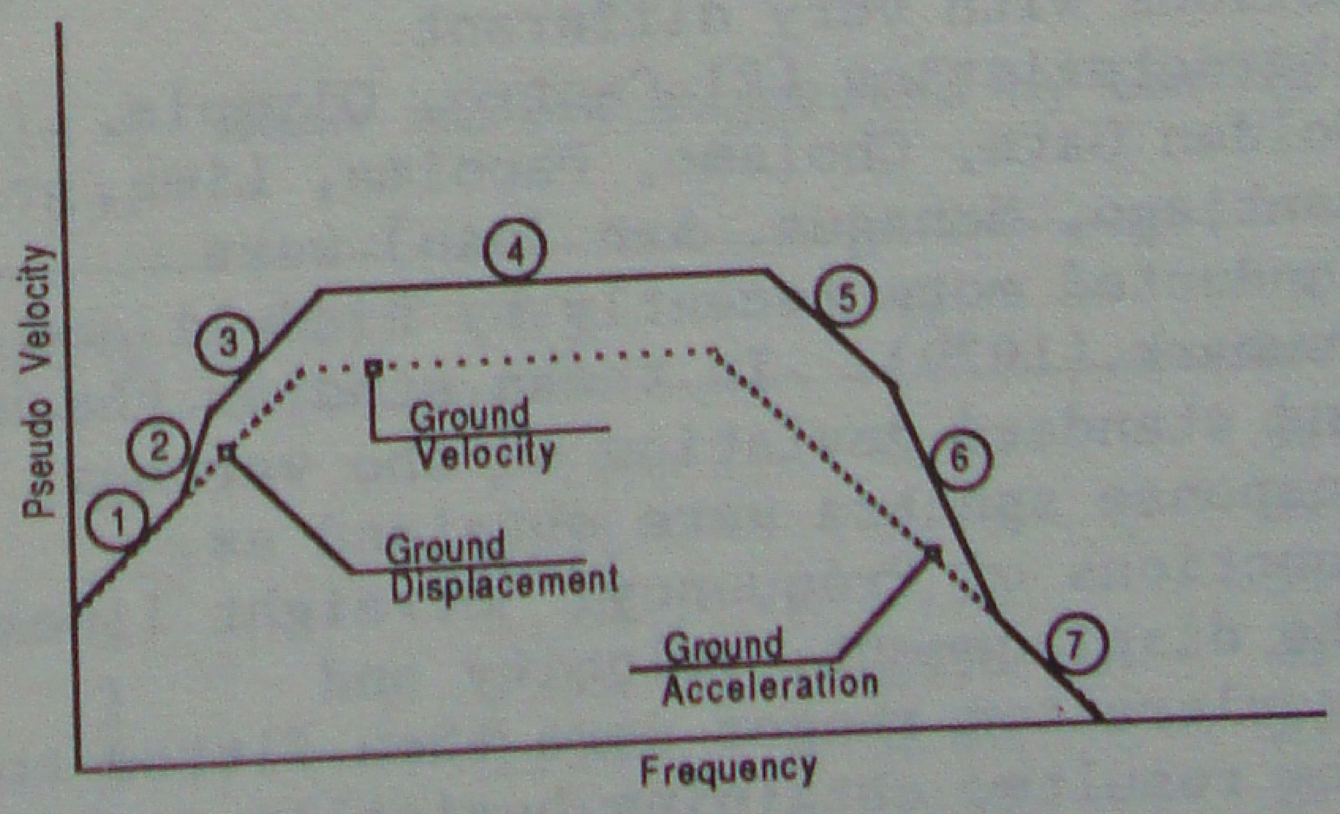

Figure 1. Newmark's smoothed spectrum.

Newmark proceeded then to define the design earthquake by three parameters: the maximum ground displacement, the maximum ground velocity and the maximum ground acceleration and suggested a set of factors by which these values had to be multiplied, as a function of damping, to obtain a smooth design spectrum. The factors originally suggested by Newmark are reproduced in Table 1. It should be noticed that these factors can be used to obtain directly the spectra from the ground motion parameters or, as is more often the case, to obtain the spectrum corresponding to an arbitrary value of damping D, given a design spectrum for another value of damping (typically 5 percent).

Table 1. Original ratios suggested by Newmark (1967) to construct elastic spectra for different values of damping.

Damping D (%)	$R_d = d_{max}/d_G$	$R_v = v_{max}/v_G$	$R_a = a_{max}/a_G$
0	2.5	4.0	6.4
0.5	2.2	3.6	5.8
1	2.0	3.2	5.2
2	1.8	2.8	4.3
5	1.4	1.9	2.6
7	1.2	1.5	1.9
10	1.1	1.3	1.5
20	1.0	1.1	1.2

These studies were continued and refined by Newmark, Hall and Mohraz (1973) who considered a set of 28 horizontal components of motion, found the individual response spectra for various values of damping and conducted a statistical analysis of the results. The mean and standard deviations of the acceleration, velocity and displacement ratios are shown in Table 2. Further studies, using different horizontal records, including motions with very different characteristics (El Centro, Olympia, Golden Gate, Cholame, Pacoima, Lima, Santiago, Managua, San Juan) were conducted more recently by Riddell and Newmark (1979). In these studies the mean and standard deviation of the various response spectra were obtained as functions of frequency. Straight lines in the displacement, velocity and acceleration ranges were then fitted to the results, obtaining basically average values over each frequency range. The recommended ratios are presented in Table 3.

Table 2. Ratios suggested by Newmark-Hall-Mohraz (1979).

Damping D (%)	R_d Mean	R_d σ	R_v Mean	R_v σ	R_a Mean	R_a σ
2	1.68	0.83	2.06	0.92	2.76	0.89
5	1.40	0.64	1.66	0.66	2.11	0.49
10	1.15	0.47	1.34	0.47	1.65	0.36

Table 3. Ratios suggested by Ridell and Newmark (1979).

Damping D (%)	R_d Mean	R_d σ	R_v Mean	R_v σ	R_a Mean	R_a σ
2	1.69	0.83	2.03	0.85	3.08	0.74
5	1.47	0.64	1.55	0.60	2.28	0.49
10	1.23	0.48	1.20	0.44	1.78	0.32

The effect of damping on elastic spectra had also been studied much earlier by Arias and Husid (1962) who concluded that for values of damping between 0.02 and 0.20 of critical and for periods in the range from 0.25 to 1.5 seconds (frequencies from 0.6 to 4 cycles per second) the ratio of the responses for damping values of D and D' could be expressed approximately by

$$\frac{S_D}{S_{D'}} = \left(\frac{D'}{D}\right)^{0.4}$$

This implies $\log S_D = -0.4 \log S_{D'} + C$ indicating that if the response were plotted at any frequency versus damping in a logarithmic scale, the points should fall on a straight line with a slope of -0.4.

Considering earthquakes as stochastic processes, Rosenblueth and Bustamante (1962) had obtained from theoretical considerations the formula

$$\log S_D = -0.45 \log(1 + 0.6Ds\omega) + \log S_o$$

where S_o is the response for an undamped system, s is an equivalent duration of the earthquake and ω is the natural circular frequency of the system.

Using five earthquakes with a total of nine components García (1970) had conducted similar studies following

Newmark's approach and comparing his results to those of Arias and Husid.

In this work the response spectra were obtained at 288 frequencies from 0.025 to 20 cps with the set of frequencies forming a geometric progression with a ratio of 1.0232. To eliminate the very sharp peaks in the response spectra of lightly damped systems it was assumed that the natural frequency of the structure might only be known within 10% and the spectral value at each frequency f was smoothed assuming a uniform window over the range 0.9 f to 1.1 f. The straight lines defining the Newmark spectrum were then obtained as envelopes to the smoothed response spectra. Finally the values of the acceleration, velocity and displacement ratios obtained for each record on the basis of these envelopes were analyzed statistically. It should be noticed that this procedure is essentially different from the one adopted later by Newmark, Hall and Mohraz and Riddell and Newmark. While in the Illinois studies the statistical analysis is performed first for all the records over all frequencies and then the straight lines are fitted averaging results over the complete frequency range of interest, García averaged each spectrum over a much narrower frequency band, obtained individual envelopes, then conducted a statistical analysis of the resulting ratios. Figure 2, taken from García's work, shows the variation of the mean values of the ratios R_a, R_v and R_d corresponding to a_{max}/a_G, v_{max}/v_a and d_{max}/d_G (maximum response acceleration, velocity or displacement divided by the corresponding peak ground motion) as a function of damping, in logarithmic scale. It can be observed that a line with a slope of -0.4 fits very well the results in the acceleration range particularly for values of damping larger than 0.05 (for smaller values of damping the results tend to show a slightly smaller slope). The same is true for the velocity range although the agreement is slightly less satisfactory. For the displacement range the slope seems to be considerably smaller particularly for values of damping less than 0.05. It should be noticed that the frequency for which the maximum displacement ratio occurs is out of the range studied by Arias and Husid.

Using, on the other hand, formulas of the type suggested by Rosenblueth, García found that excellent agreement could be provided by the expressions

$$R_a = 9\ (1 + 325\ D)^{-0.4}$$

$$R_v = 4.5\ (1 + 125\ D)^{-0.4}$$

$$Rd = 2.1\ (1 + 22\ D)^{-0.4}$$

which would correspond to

$$\log R_D = -0.4 \log (1 + kD) + \log R_o$$

with R_o = 9 for the acceleration range, 4.5 for the velocity range and 2.1 for the displacement range and the constant k equal to 325, 125 and 22 for each range.

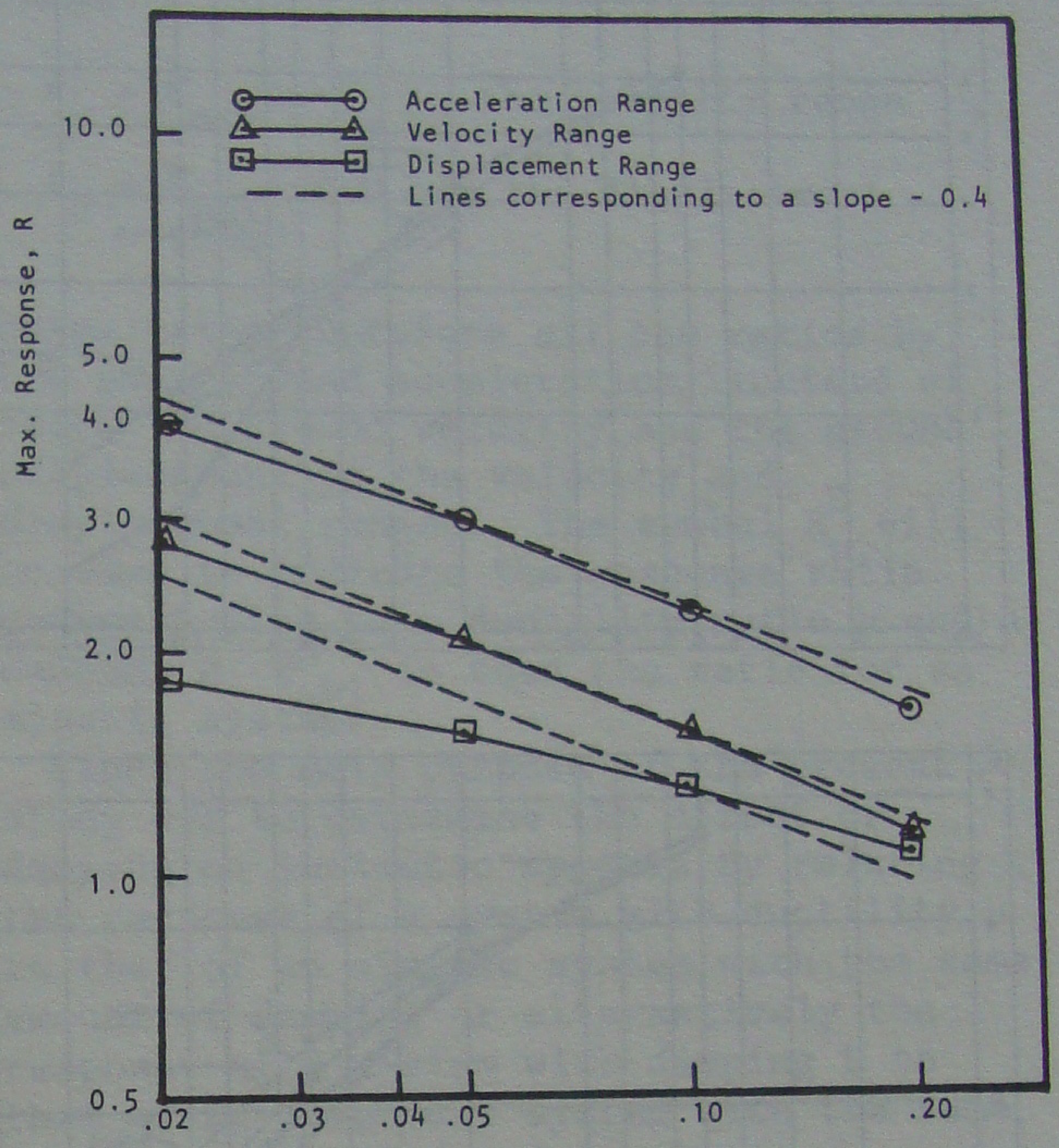

Figure 2. Variation of R_a, R_v and R_d with damping (Garcia, 1970).

Mean factors corresponding to these formulae are shown in Table 4. These values are close to those Newmark had originally recommended except for undamped systems and for the larger values of damping (where García's results are somewhat larger).

Table 4. Mean ratios suggested by García (1970).

Damping D (%)	R_d	R_v	R_a
0	2.10	4.50	9.00
2.5	2.01	3.70	6.10
1	1.94	3.25	5.01
2	1.81	2.71	4.00
5	1.56	2.02	2.88
10	1.32	1.59	2.20
20	1.07	1.22	1.67

Figure 3 shows a comparison of the ratios recommended by Newmark, Newmark, Hall and Mohraz, Riddell and Newmark and García in the acceleration, velocity and displacement ranges. It is interesting to notice that in spite of the differences in the earthquake records used for these studies the trends (variation of the ratios with damping) are very similar for the last three studies and almost identical for García's and Riddell's.

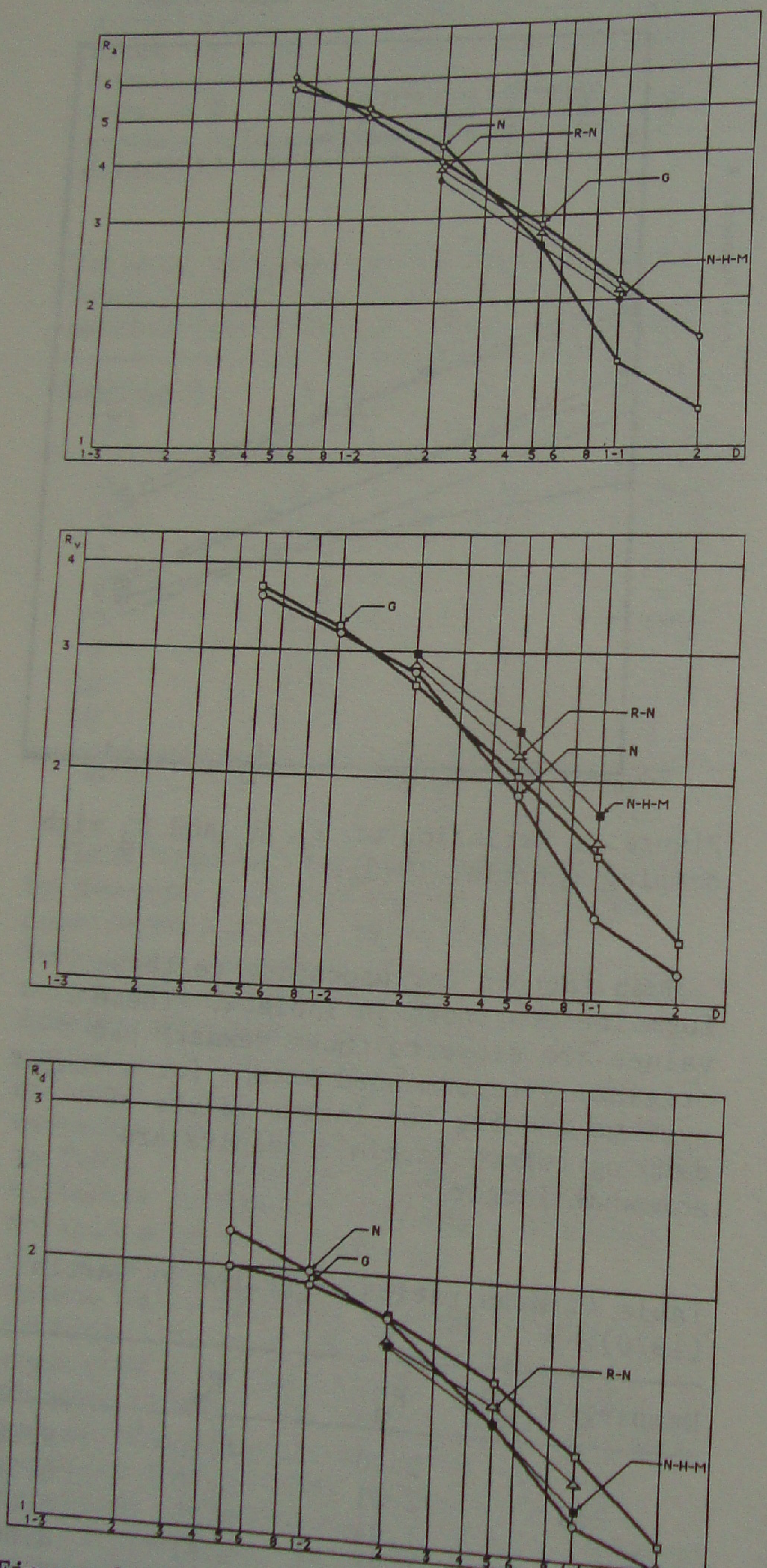

Figure 3. Variation of R_a R_v R_d with damping. Comparison of results of various studies G - Garćia, N - Newmark, N-H-M - Newmark-Hall-Mohraz, R-N - Riddell-Newmark.

García's mean values are very close to the mean plus one standard deviation of Riddell and Newmark for the acceleration and velocity ranges which are the ones of main interest for nuclear power plants and regular buildings (they are slightly larger in the acceleration range and slightly smaller in the velocity range). In the displacement range, on the other hand, García's values are closer to the mean values of Riddell and Newmark (slightly larger), although the effect of damping (slope of the lines) is still almost identical.

All these results are of value when designing structures which are assumed to remain nearly elastic under the design earthquake (such used to be the case for nuclear power plants, for instance). Most regular buildings will undergo inelastic deformations when subjected to strong earthquake motions, a fact implicitly recognized in design codes and accepted in present design philosophy.

Inelastic response spectra for elasto-plastic systems with zero and ten percent damping were presented by Blume, Newmark and Corning (1960) for the NS component of the 1940 El Centro earthquake and ductility ratios of 1, 1.25, 2 and 4. The ductility ratio was defined in this study as the ratio of the maximum distortion of the SDOF system to its yield distortion.

Newmark and Hall (1973) suggested rules to construct inelastic spectra for different levels of ductility and elasto-plastic systems with 5 percent damping, from the corresponding elastic spectrum. It is important to notice that for inelastic systems a single spectrum will no longer provide the values of acceleration, velocity and displacement. For a given ductility ratio μ the displacement spectrum will be equal to the acceleration spectrum multiplied by μ and divided by the square of the natural circular frequency ω^2. Newmark's rules to obtain the acceleration (or pseudo acceleration) spectrum consist of dividing the elastic spectrum by μ in the displacement and velocity ranges (actually ranges 1 through 4) and by the square root of 2μ-1 in the acceleration range (range 5). The inelastic spectrum is equal to the elastic spectrum in the 7th range independently of the ductility.

Studies by Sehayek (1976) using again the five earthquakes (nine components) of García's work showed that Newmark's rules applied very well to the average results for these earthquakes and elasto-plastic systems with 5 percent damping, although they do not provide a uniform degree of conservatism over all the frequency ranges.

A basic question that remains to be answered is whether these rules would apply equally to elasto-plastic systems with a different value of initial damping. It should be noticed in this respect that the specified value of damping represents the energy dissipation that would take place under small deformations or due to the action of nonstructural components since the hysteresis losses are already accounted for in the nonlinear model.

The purpose of this work is to assess the effect of initial damping on inelastic spectra for single degree of freedom elasto-plastic systems, and to suggest rules to construct these spectra from elastic spectra.

2 METHOD OF SOLUTION

An elasto-plastic SDOF system with damping values of zero, two, five, ten and twenty percent was used in the study. The system was subjected to five different earthquakes with a total of nine components, the same records used by García and Sehayek. These represent moderate duration earthquakes recorded on firm ground. The earthquakes were: El Centro 1940 NS and EW components; Taft 1952 N69W and S21W; Olympia 1949 N10E and N80E; Helena 1935 NS and EW; and Golden Gate S80E.

For a system with mass M, damping D, natural frequency ω and yield strength F_y the time history of the system's response was determined through numerical integration for each input earthquake. From the time history, the maximum response displacement y_{max} and the corresponding ductility ratio were obtained. The numerical integration scheme used was based on the Central Difference Formula. Two different procedures were used: in the first one once a value of the ductility ratio was computed the yield strength was automatically modified until the desired value of μ was obtained. In the second one results were obtained for different values of the yield force and the resulting ductility ratios were plotted versus $M\ddot{U}_G/F_y$ or $\ddot{U}_G/\ddot{U}_{max}$. The value of this ratio corresponding to the desired ductility ratio was then obtained by interpolation. Results from these two procedures were almost identical. The plots of R_a/R_v R_d were then averaged for all the earthquakes at each frequency considered. Finally, straight lines were drawn over the three ranges considered by smoothing the average spectra and enveloping them.

Dimensionless response ratios were again computed for the acceleration, velocity and displacement ranges. In this case, however, it was found more convenient, due to the fact that inelastic spectra were considered, to define

$R_a = \ddot{U}_{max}/\ddot{U}_G$ in the acceleration range

$R_v = \ddot{U}_{max}/\omega\cdot\ddot{U}_G$ in the velocity range

$R_d = \ddot{U}_{max}/\omega^2\cdot\ddot{U}_G$ in the displacement range

normalizing therefore all the ratios by the peak ground acceleration, instead of using the ground velocity and the ground displacement in the velocity and displacement ranges. The symbol R^D_μ will be used to indicate the response ratio corresponding to a ductility ratio μ and a damping D. $R^D_{\mu=1}$ is then the ratio for an elastic system.

Since the main purpose of the present study was to determine the effect of damping on inelastic spectra by relating the response of a system with ductility μ to that of an elastic system with the same amount of damping or alternatively the response of a system with damping D to that of an identical system with the same ductility but 5% damping, the normalization factor is of no consequence and disappears when the ratios of the two responses are computed.

3 EFFECT OF DAMPING ON ELASTIC RESPONSE SYSTEM

The effect of damping on elastic spectra was studied first in order to calibrate the procedure and compare the results with those reported in the literature and discussed earlier. It should be noticed that although the earthquake records and the overall procedure were the same used by García the order in which the smoothing of the spectra and the averaging of the envelopes was performed was the opposite. As a result the values obtained had some slight differences with those reported by García. They were still very close to these ones and to the mean plus one standard deviation results of Riddell and Newmark for the acceleration and velocity ranges and their mean results for the displacement range. As noted in discussing García's results the exponent in a simple formula like that suggested by Arias and Husid seemed to be different for values of damping larger and smaller than 0.05. To make this difference more apparent the results are presented in Figure 4 in terms of the ratio of the value of R corresponding to a damping D to the value of R for D = 0.05.

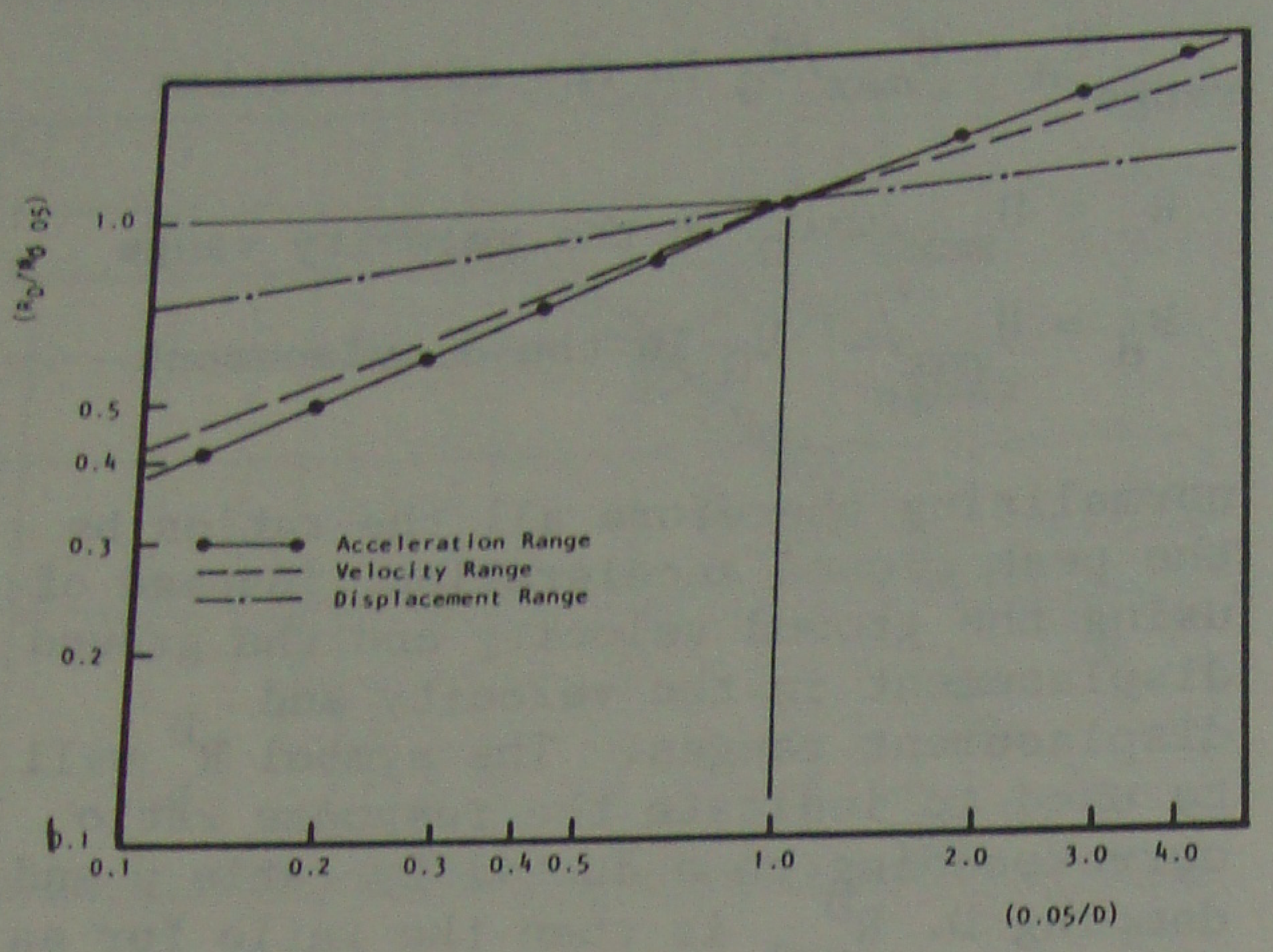

Figure 4. Plots of response versus damping for ductility of one and for the acceleration, velocity and displacement ranges.

Writing then

$$R^D = \left(\frac{0.05}{D}\right)^{\alpha} R^{.05}$$

the resulting values of α are as shown in Table 5.

Table 5. Exponent α for elastic system.

Damping	Values of α		
	Acceleration Range (α_a)	Velocity Range (α_v)	Displacement Range (α_d)
0.02-0.05	0.343	0.286	0.080
0.05-0.20	0.415	0.371	0.130

4 EFFECT OF DAMPING ON INELASTIC RESPONSE SPECTRA

As pointed out earlier, Newmark studied the effect of ductility on the response using an elasto-plastic SDOF system with 0.05 damping subjected to the El Centro earthquake. Newmark's rules for obtaining inelastic response spectra for 0.05 damping and ductility μ are the following:

- for velocity and displacement ranges

$$R^{0.05}_{\mu=1}/R^{0.05}_{\mu} = \mu$$

- for acceleration range

$$R^{0.05}_{\mu=1}/R^{0.05}_{\mu} = \sqrt{2\mu-1}$$

Values of the ratio of maximum elastic response to inelastic response for a given value of damping ($R^D_{\mu=1}/R^D_{\mu}$) obtained in this work were plotted versus ductility μ for all values of damping considered and for the three ranges (see Figures 5a, b, and c). Newmark's lines are also shown in these figures. Comparing the plots obtained in this study with Newmark's, it can be observed that Newmark's curves almost coincide with the response curves for 0.05 damping in all three frequency ranges. Hence, it was concluded in this work that Newmark's rules could be used to obtain the inelastic response spectra for 0.05 damping and for a given value of μ knowing the elastic response for 0.05 damping. For smaller values of damping, however, the reduction in the spectral response due to inelastic behavior (ductility μ) is larger than suggested by Newmark's rules, while it is smaller for larger values of damping. This is

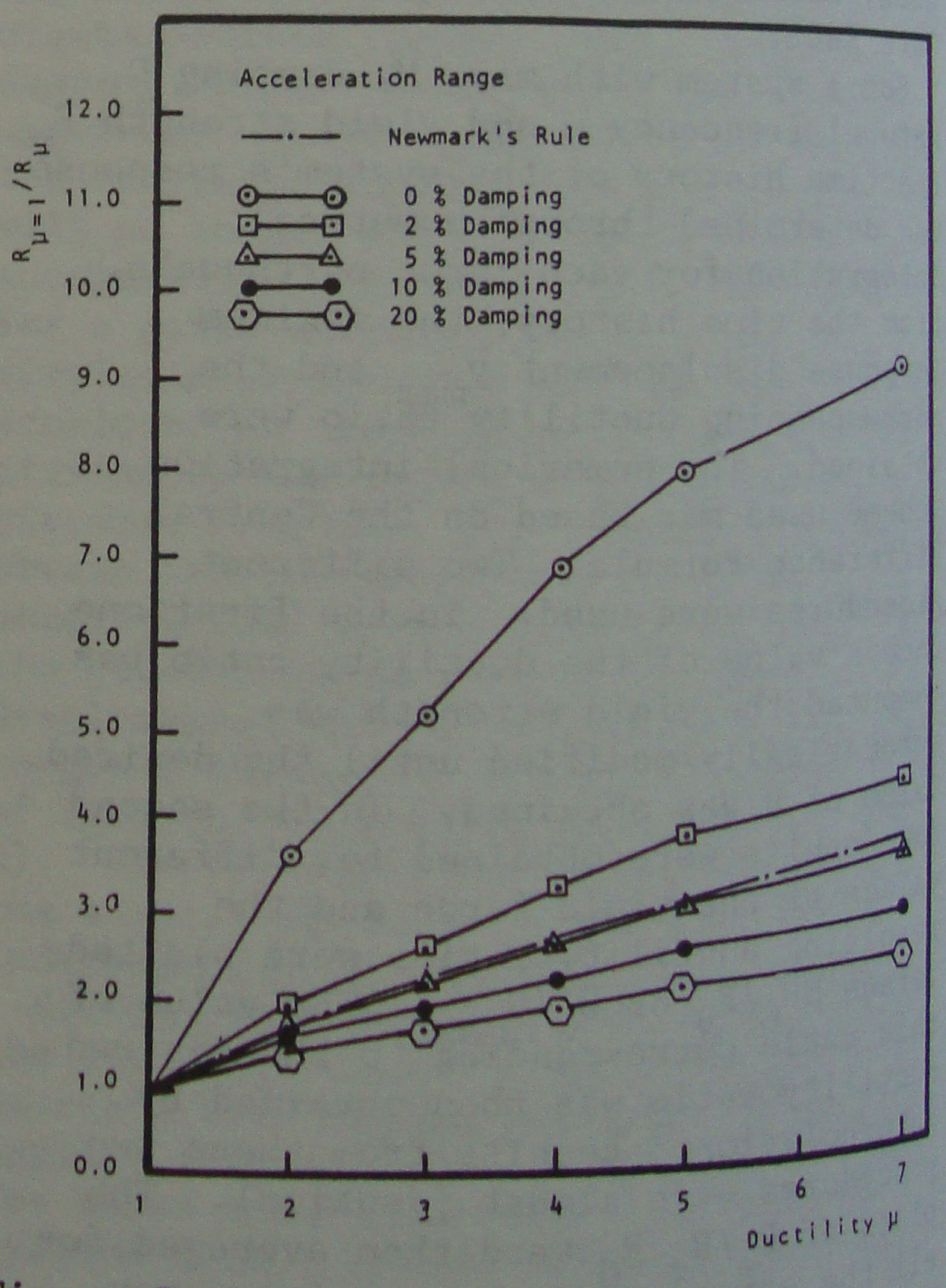

Figure 5a. Acceleration range.

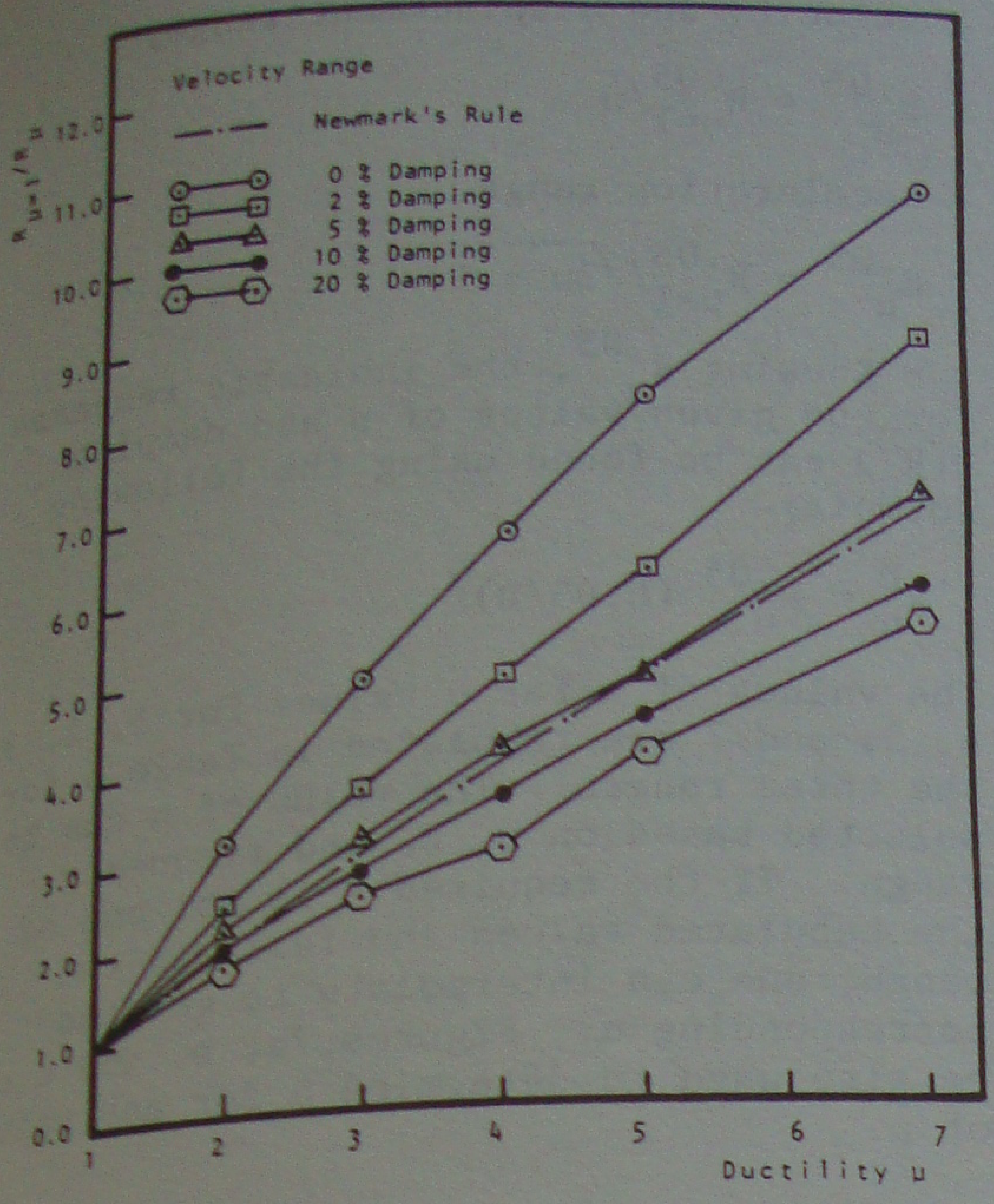

Figure 5b. Velocity range.

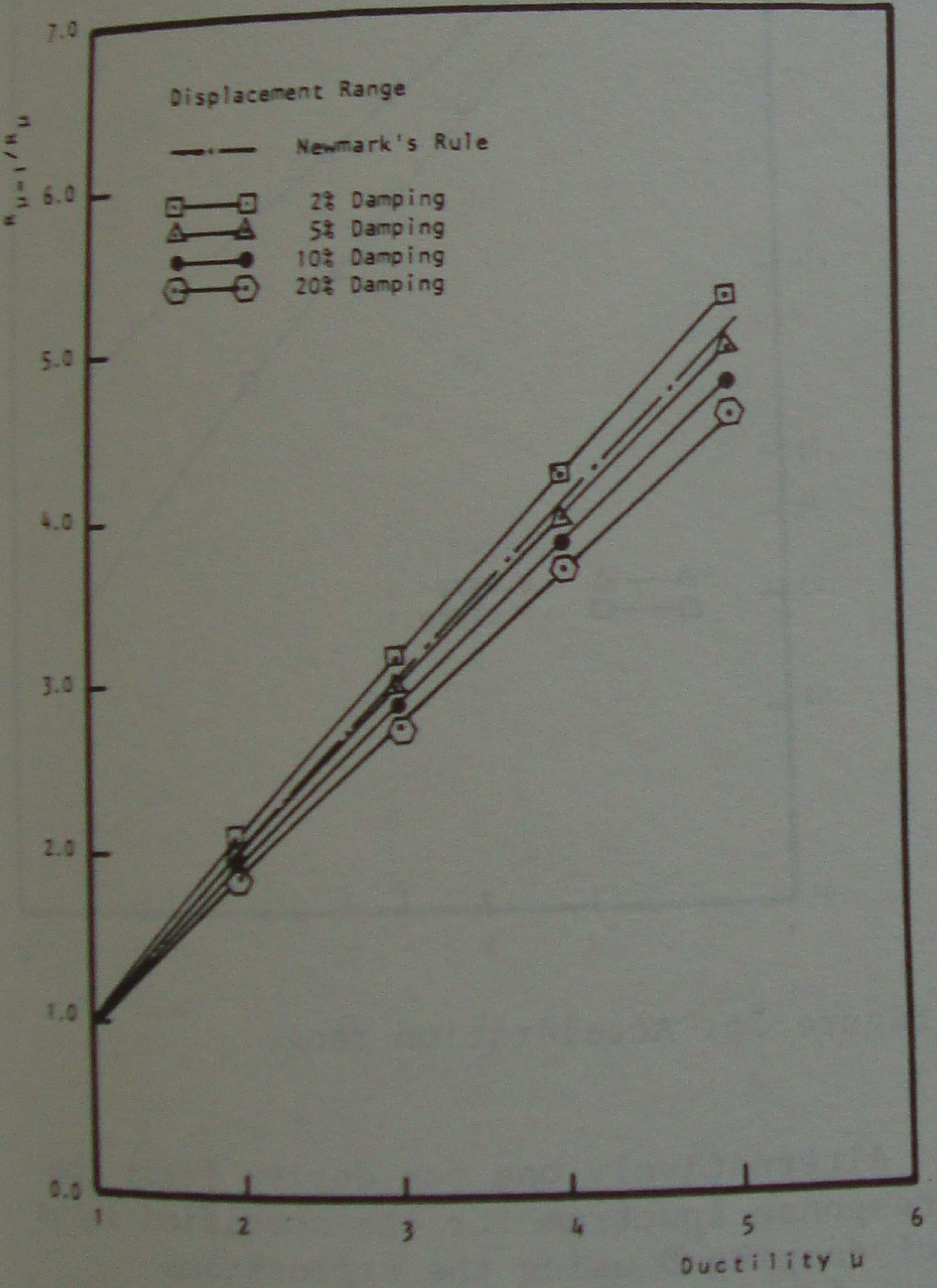

Figure 5c. Displacement range.

Figure 5. Plots of the ratio of elastic response to inelastic response versus ductility.

understandable if one considers the effect of inelastic behavior as a reduction in the natural frequency of the system and a corresponding increase in damping (due to energy dissipation by hysteresis). Clearly the effect of this additional damping is much more significant for a system which has initially a very small value of D than for systems with substantial initial damping.

The inelastic response values for 0.05 damping were used in this study as a basis for the determination of the effect of damping on the inelastic response spectra. For a given ductility μ, the values of the ratio of the response for a given amount of damping to that for 0.05 damping ($R_\mu^D/R_\nu^{.05}$) were plotted versus damping ratio (0.05/D) on a logarithmic scale and in the form: $R_\mu^D/R_\mu^{.05} = (0.05/D)^\alpha$. Three plots were obtained for each μ corresponding to the three frequency ranges (for example, see Figure 6 for μ = 2). It can be seen from this figure that each plot consists of two straight line segments each with a different slope α. The value of α of the segment in the damping range 0.02 - 0.05 is smaller than the one in the damping range of 0.05 - 0.20. This indicates that, for a given value of ductility μ, the rate of decrease of the response with increasing amount of damping is slower in the damping range 0.02 - 0.05 than in the range 0.05 - 0.20.

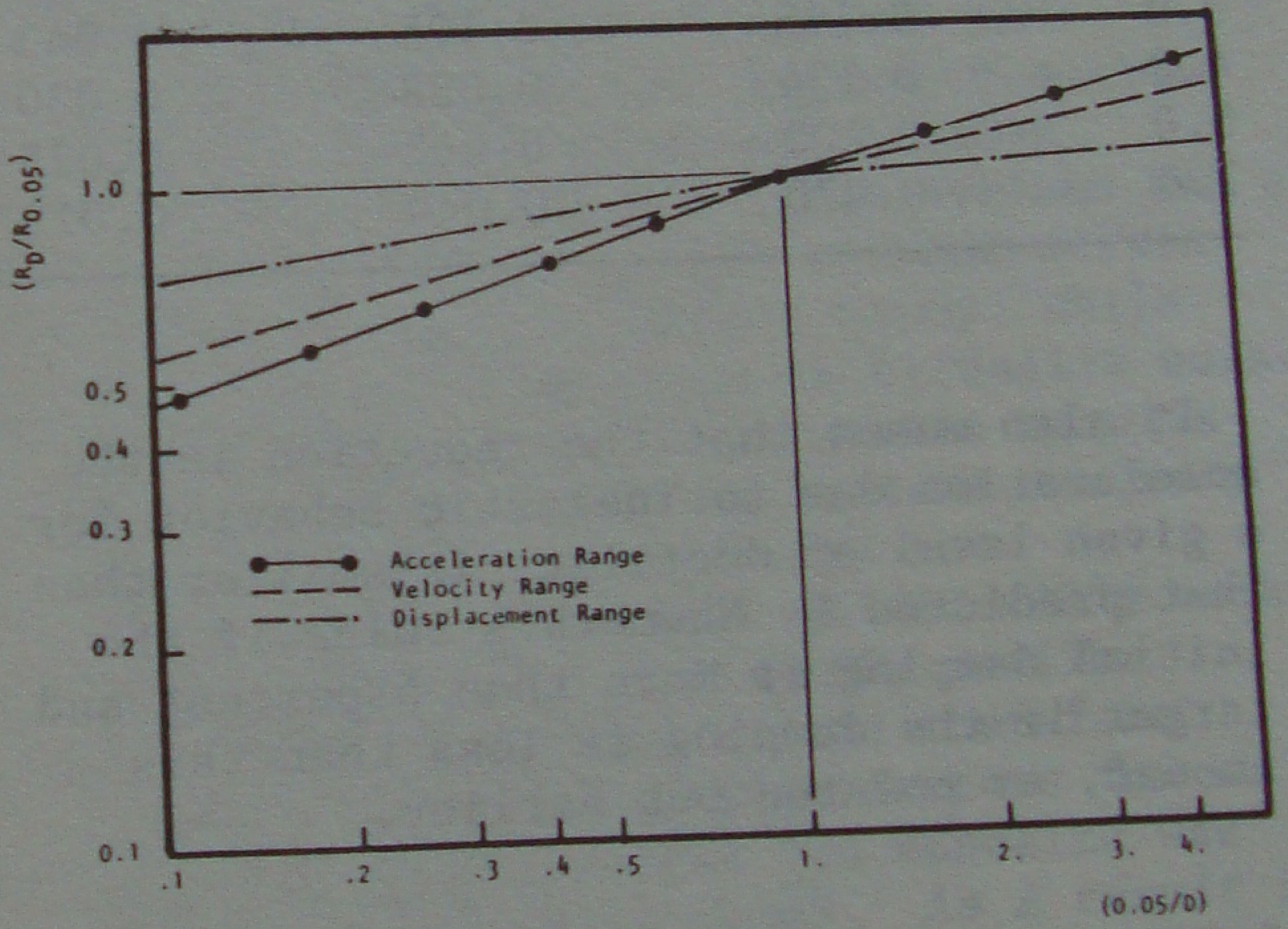

Figure 6. Plots of response versus damping for ductility of two and for the acceleration velocity and displacement ranges.

Values of α as a function of μ are presented in Table 6 for the acceleration, velocity, and displacement ranges each with two damping ranges. Looking at the values of α in this table for any of the three ranges, it can be seen that α decreases with increasing μ. This means that the effect of damping on the inelastic response decreases with increasing μ. This behavior can be explained by the fact that as μ increases, the hysteretic damping increases and dominates over the viscous damping D which in turn makes the effect of the latter on the response smaller.

Table 6. Values of the exponent α, as a function of ductility μ, for the acceleration, velocity, and displacement ranges.

μ	(0.05/D) < 1		
	α_a	α_v	α_d
1.0	0.415	0.371	0.130
2.0	0.333	0.273	0.106
3.0	0.252	0.196	0.088
4.0	0.201	0.155	0.071
5.0	0.163	0.138	0.065
7.0	0.125	0.106	0.055

μ	(0.05/D) > 1		
	α_a	α_v	α_d
1.0	0.343	0.286	0.080
2.0	0.262	0.196	0.065
3.0	0.195	0.125	0.043
4.0	0.134	0.088	0.030
5.0	0.102	0.063	0.025
7.0	0.070	0.040	0.018

It also means that the reduction in the acceleration due to inelastic behavior for a given level of ductility is smaller than that predicted by Newmark's rules if the initial damping is more than 5 percent and larger if the damping is less than this amount, as pointed out earlier.

5 DETERMINATION OF INELASTIC RESPONSE SPECTRA FOR SPECIFIED VALUES OF μ AND D

A procedure to obtain inelastic response spectra for given values of damping D and ductility μ if the elastic response values for the three ranges and 0.05 damping ($R^{.05}_{\mu=1}$) are given, can then be summarized as follows:

- Using Newmark's rules, the inelastic response values for 0.05 damping and ductility μ can be found ($R^{.05}_{\mu}$):

velocity and displacement range,

$$R^{.05}_{\mu} = R^{.05}_{\mu=1}/\mu$$

acceleration range,

$$R^{.05}_{\mu} = R^{.05}_{\mu=1}/\sqrt{2\mu - 1}$$

- Knowing $R^{.05}_{\mu}$, the inelastic response for the given values of μ and damping D(R^{D}_{μ}) can be found using the following formula:

$$R^{D}_{\mu} = R^{.05}_{\mu}\ (0.05/D)^{\alpha}$$

The values of α for μ values for 1, 2, 3, 4, 5, and 7 are tabulated in Table 6 for the three ranges. The value of α can be selected based on μ, D, and frequency range. If the required μ is not one of the tabulated values and falls in the range, one can interpolate to get the corresponding α. Figures 7a, b, and c can be also used to determine α for any value of μ.

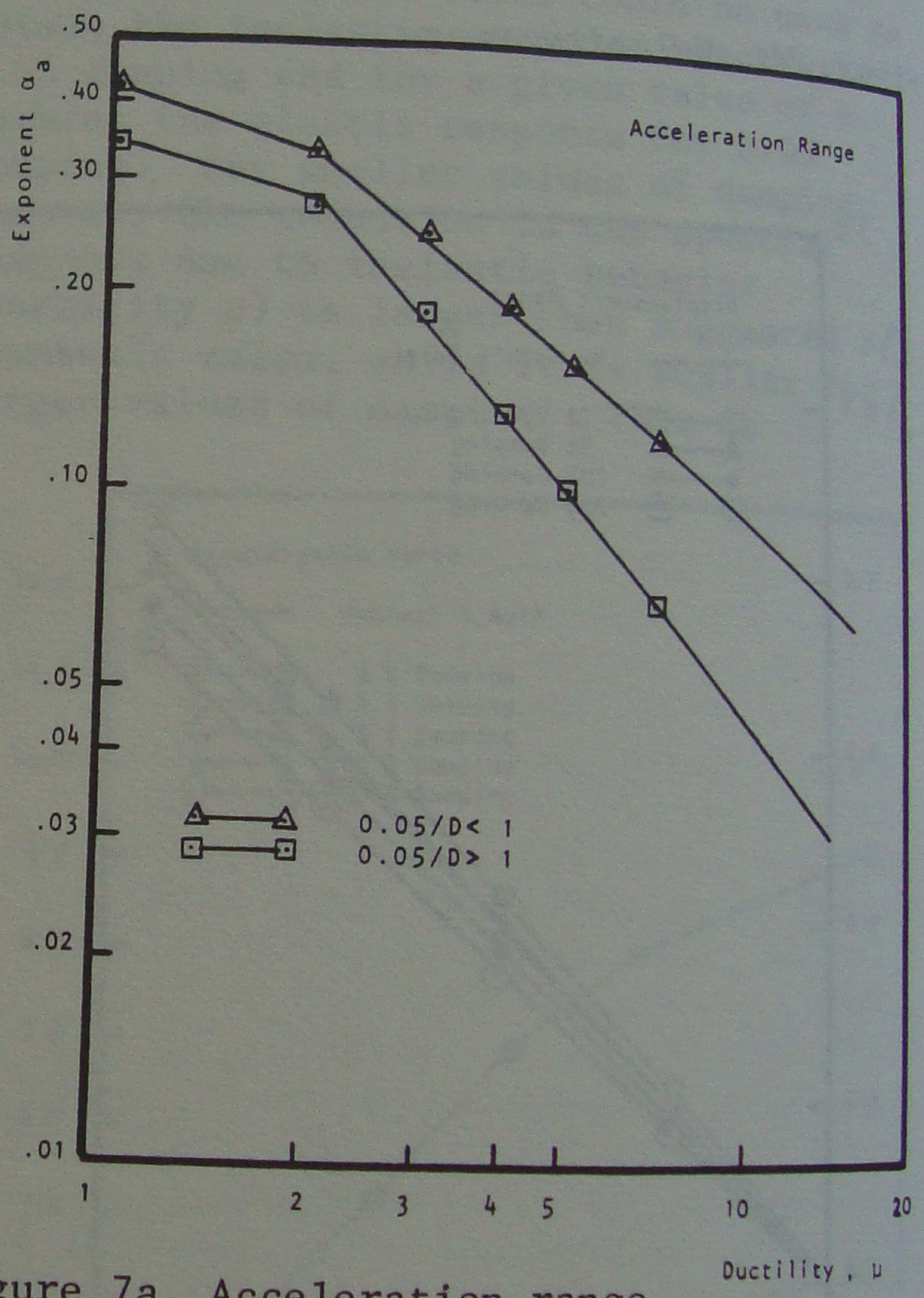

Figure 7a. Acceleration range.

Alternatively one can derive first the response spectrum for the specified value of damping D using the expressions provided by García (1970) or the factors suggested by Riddell and Newmark (1979), then apply reduction factors depending on the ductility as shown in Figure 5a, b, and c.

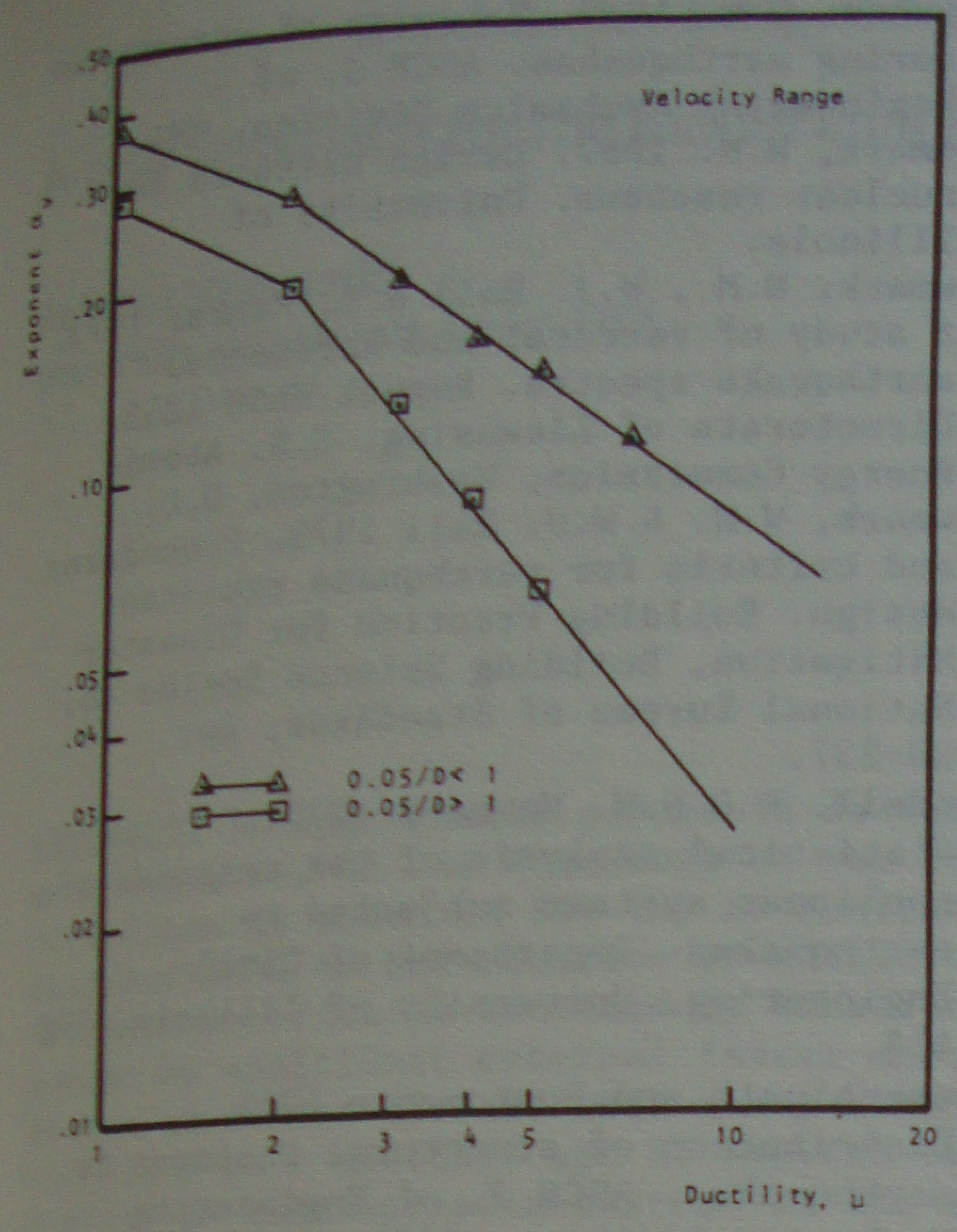

Figure 7b. Velocity range.

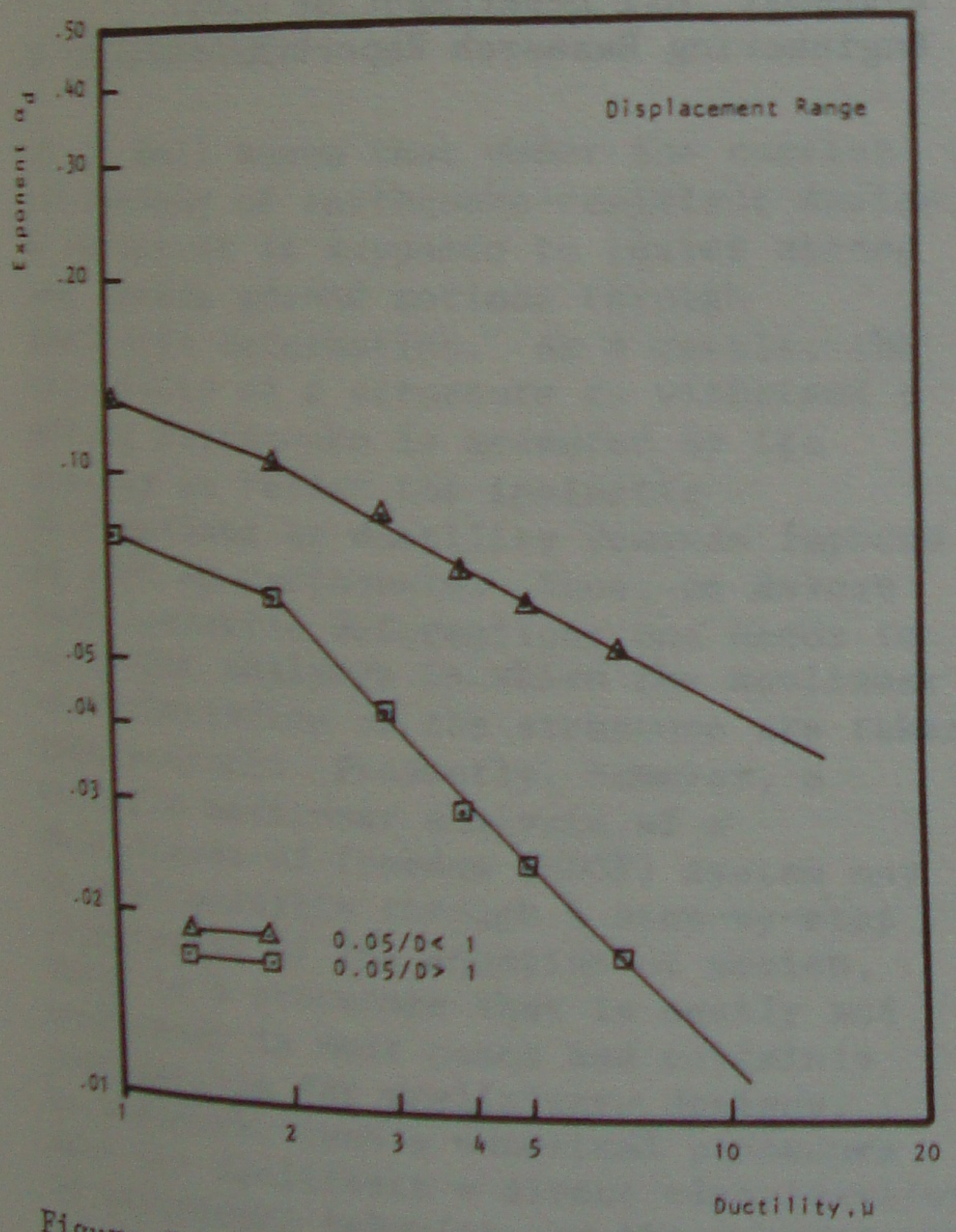

Figure 7c. Displacement range.

Figure 7. Plots of the exponent versus ductility for damping ranges less and greater than 0.05.

Inelastic spectra for elasto-plastic systems with 2, 5 and 10% damping were also obtained by Riddell and Newmark (1979). Their study shows the same trends reported here although the actual values are slightly different due to the variations in the motions considered and the procedure used. Their reduction factors by which the elastic spectra have to be divided to obtain the inelastic spectra are larger than those of the present study in the displacement range (by 10 to 15%) and in the velocity range (by 5 to 10%) for small ductilities. They tend to be smaller on the other hand for larger values of the ductility. Both studies show on the other hand that these reduction factors are about 1.05 times those corresponding to 5% damping for systems with D = 0.02 in the displacement range and 0.95 times the 5% damping factors for D = 0.10. These values vary from 1.10 to 1.25 (increasing with ductility) for D = 0.02 in the velocity and acceleration ranges and from 0.94 to 0.85 (decreasing with increasing ductility) for D = 0.10. The effect of damping is therefore more pronounced in the velocity and acceleration ranges than for small frequencies (displacement range).

Similar studies have been conducted by the authors for different nonlinear systems with stiffness degradation, pinching and strength degradation (1982). Riddell and Newmark considered also bilinear and stiffness degrading systems but only with 5% damping. A basic difference is that all these systems had a perfectly plastic range, without strain hardening, in the present study, while they had a second slope in Riddell's work. Both studies concluded that the results for stiffness degrading systems are very close to those reported for the elasto-plastic models. In Riddell's work they tended to have slightly smaller ductility requirements (and therefore slightly larger reduction factors for a given ductility) while the opposite was true in the present study. As a result it was found that the decrease in the effect of the initial damping was slightly less pronounced when there was stiffness degradation, and even less when there was additional pinching, since the internal damping generated by the inelastic behavior for a given ductility level was somewhat smaller than for the elastic-perfectly plastic systems (the exponents α were therefore slightly larger for these systems).

6 SUMMARY

The effect of damping on elastic and inelastic spectra for elasto-plastic systems has been investigated by considering five earthquakes with a total of nine records. The results for elastic systems are slightly different from those previously reported by other researchers (particularly García, who worked with the same set of earthquakes) due mainly to the differences in the averaging and smoothing processes used. These differences are not, however, significant. The results for elasto-plastic systems indicate clearly that as the ductility increases the effect of the initial damping, corresponding to energy dissipation at low levels of strain or by nonstructural components, becomes less and less important. For a stiff structure whose natural frequency places it in the acceleration range the reduction in the value of the response (spectral) acceleration for a ductility of 4 would be 2.65 if the initial damping was 5% but only 2.27 if the damping were 10%. The reduction factor would increase however to 3.23 if the system had only 2% damping. The corresponding reduction factors would be 4, 3.44 and 4.83 in the velocity range and 4, 3.78 and 4.25 in the displacement range.

It should be pointed out finally that these rules, as well as Newmark's ones, are only applicable to smoothed design spectra. Applying them to the results of elastic analyses with an actual earthquake record, to estimate the inelastic response, could give very erroneous answers (generally unconservative).

REFERENCES

Al-Sulaimani, G.J. 1982. Inelastic response of degrading and nondegrading systems under seismic actions. Ph.D. Dissertation. University of Texas at Austin, Department of Civil Engineering.

Arias, A. & R. Husid 1962). Influencia del amortiguamiento sobre la respuesta de estructuras sometidas a temblor. Revista del IDIEM, Vol. 1, No. 3.

Blume, Newmark, and Corning 1960. Design of multistorey reinforced concrete buildings for earthquake motions. Portland Cement Association.

García, F. & J.M. Roesset 1970. Influence of damping on response spectra. MIT Department of Civil Engineering Research Report R70-4.

Housner, G.W. 1959. Behavior of structures during earthquakes. ASCE J. of Engineering Mechanics Division, EM4.

Newmark, N.M. 1967. Design criteria for nuclear reactors. University of Illinois.

Newmark, N.M., W.J. Hall & B. Mohraz 1973. A study of vertical and horizontal earthquake spectra. Report WASH-1255, Directorate of Licensing, U.S. Atomic Energy Commission, Washington, D.C.

Newmark, N.M. & W.J. Hall 1973. Procedures and criteria for earthquake resistant design. Building Practice for Disaster Mitigation, Building Science Series 46, National Bureau of Standards, pp. 20-237.

Riddell, R & N.M. Newmark 1979. Statistical analysis of the response of nonlinear systems subjected to earthquakes. Department of Civil Engineering, University of Illinois, SRS 468.

Roesenblueth and Bustamante 1962. Distribution of structural response to earthquakes. ASCE J. of Engineering Mechanics Division, EM3.

Sehayek, S. 1976. Effect of ductility on response spectra for elasto-plastic systems. MIT Department of Civil Engineering Research Report R76-42.